Inmetro Advanced School on Uncertainty of Measurement, December 2007			
Item	**Date**	**Description**	**Notes**
1	10/12 W. Woeger	Introduction	Probabilistic basis of uncertainty evaluation. Stages of uncertainty evaluation. Model-based uncertainty evaluation. PDFs and distribution functions. Random variables and quantities. Summary statistics. Coverage intervals. Particular distributions
2	10/12 W. Woeger	Probability as degree of rational belief. PDFs, random variables, expectation, standard deviation, variance, coverage interval, etc. (brief reminder)	Options: (1) omit (cover in introductory course), (2) introduce at this stage, (3) introduce as needed. Stress that probability interpretation different from frequency (information-based probability)
3	10/12 W. Woeger	Interpretation of parameters of PDF in metrology (brief reminder)	(Best) estimate. Standard uncertainty. Uncertainty matrix. Coverage intervals. Biased estimates. Multivariate case.
4	10/12 W. Bich	Work and objectives of JCGM/WG1	JCGM. GUM evolution. GUM Supplements. Other documents. Electronic GUM. Revised GUM
5	10/12 W. Bich	Model-based uncertainty evaluation	Main stages of uncertainty evaluation. Modelling stage of uncertainty evaluation. PDFs. Modelling example: how long is a piece of string? Calculation stage. Matrix formalism
6	10/12 W. Bich	Available information concerning input quantities	Results from the application of Principle of maximum entropy. Bayes' theorem. Information-based assignment of distributions
7	10/12 W. Woeger	(Joint) PDFs for set of quantities	Covariance as parameter. Uncertainty matrix and its properties
8	10/12 W. Woeger	Models with common effects and the consequent logical correlation	E.g., pair of models, $Y_1 = X_1 + X_2$, $Y_2 = X_1 + X_3$, used to deduce $u(y_1)$, $u(y_2)$ and $u(y_1, y_2)$, given $u(x_1)$, etc., and possibly $u(x_1, x_2)$, etc. Important consideration to guarantee internal consistency of evaluation method
9	10/12 W. Woeger	Aspects of frequentist approach If at all: After Bayes' theorem	Included only to indicate its inadequacies, e.g., inability to account for logical correlation. (Example.)
10	11/12 W. Woeger	PME, and its use to assign PDFs in minimal way given information concerning quantities	GUM Supplement JCGM 101

11	10/12 W. Woeger	Bayes' theorem	Presentation depending on introductory week (Ignacio Lira)
		Application of Above Concepts to Evaluation of Measurement Uncertainty	
13	11/12 W. Bich	Model classification. Introduce multivariate models, including complex models, and implicit models	Classification of models. Univariate or multivariate. Explicit or implicit. Real or complex. Examples. Matrix calculation. Matlab
14	11/12 W. Bich	Multi-stage models/sub-models	Multi-stage models. Sub-models. Building up a model. Or decomposition of a model. GUM: internal consistency. Example: calibration curves. GUM H1: Gauge-block calibration
16	11/12 W. Woeger	The propagation of distributions	The propagation of distributions. GUM Supplement JCGM 101. Other analytic or numerical methods [GUM G.1.5]
17	11/12 W. Bich	Implementing the law of propagation of uncertainty	Only estimates and uncertainties (uncertainty matrices) are propagated. Forming sensitivity coefficients often the hardest part. Proposed way of doing so. Avoids the numerical subtraction associated with finite-difference approximation of the GUM.
18	11/12 W. Bich	GUM expanded uncertainty	GUM uncertainty framework approach. Central limit theorem
19	11/12 W. Bich	GUM uncertainty framework. Limitations	Inputs and outputs for the GUM uncertainty framework. Components: law of propagation of uncertainty (LPU), central limit theorem (CLT). Assumptions. Step-by-step procedure. Flow diagram
20	12/12 W. Woeger	Analytic solutions	Particular analytic methods. Approximate analytic methods. Single dominant effect. Examples Case study: neutron ambient dose equivalent measurement
21	12/12 W. Bich	Monte Carlo method (MCM) as faithful implementation of the propagation of distributions	Monte Carlo method (MCM): the simplest, most effective and efficient approach. Propagates discrete representations of PDFs for input quantities through measurement model. Evaluates summary information for output quantity in terms of this discrete representation. Inputs and outputs. Based on repeated sampling from PDFs for input quantities
22	12/12 W. Bich	Practical MCM issues	Practical MCM issues. Drawing from any distribution function. Drawing from particular distributions. Number of Monte Carlo trials. Numerical accuracy. Timing. Validation of GUM uncertainty framework

23	12/12	Relationship between expanded uncertainty and coverage intervals	Comparability generally only for Gaussian PDF. Symmetric PDFs
24	12/12	Relationship of above to Bayesian reasoning/methods.	More on Bayesian methods
25	12/12	Reconciliation of GUM use of best estimates and uncertainties with probabilistic approach through maximum entropy considerations	"Reassurance" that conventional GUM (LPU) is often adequate. Assistance with alignment of approaches
26	12/12 W. Woeger	Prior for output quantity	Relate to metrologist's knowledge, e.g., chemical concentration in [0 %, 100 %]. Recalibration problem: not treated in GUM
27	12/12 W. Woeger	Adaptation of propagation of distributions to use prior	Thence fully Bayesian. Elster paper
28	12/12	According adaptation of MCM	Elster paper
29	12/12	Bayesian implementations	MCMC, WinBUGS, etc.
		Further Applications	
30	13-14/12		E.g., calibration, least squares fitting, key comparison data evaluation, conformance testing; run live examples. For at least one example, give formulation and calculation details. A beautiful example for a full Bayesian analysis is the evaluation of the PDF for the emission rate of gamma quanta sent by an astronomical source, when the observed counts are known to contain background events (see e.g. Gregory, P.C.: Bayesian logical data analysis for the physical science, Cambridge Univ. Press, Cambridge, 2005)
31	14/12 W. Bich	Comparison of approaches	Comparison of approaches. Run live examples
		Markov Chain Monte Carlo (MCMC)	
32	13/12 D. Gamerman	Introduction to Bayesian modeling. Approximate approaches to inference. Introduction to stochastic simulation.	
33	13/12 D. Gamerman	Markov Chain Monte Carlo (MCMC). Application examples. Illustration via WinBUGS.	
		Further Bayesian Applications	
34	14/12 I. Lira	Bayesian inference from comparison data.	With special emphasis on inconsistent data.
35	14/12 I. Lira	Probabilistic and least-squares inference of the parameters of a straight-line model.	Presents a specialized application of some of the concepts learned in the Advanced School.

		Further Applications	
36	14/12 W. Bich	The uncertainty of a calibrated instrument: instrument noise and uncertainty from calibration. The best estimate in a non-linear model. Some applications of least squares: constraints, certain and with uncertainty.	
		Round table on the evolution of the GUM	
37	14/12 W. Bich W. Woeger I. Lira	Discussions about the present and future of the GUM. The new edition of the VIM. Measurement as inference. The role of information. The increased use of numerical methods.	This time can also be used for answering specific questions of the attendees.