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AN APPROACH BASED ON ASYNCHRONOUS SAMPLING AND FAST HARTLEY TRANSFORMS FOR EVALUATING HARMONICS OF PERIODIC SIGNALS WITH NEGLIGIBLE LEAKAGE

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Abstract

An algorithm based on Fast Hartley Transform (FHT) that uses a digital voltmeter for measuring the harmonic parameters of low-frequency signals with negligible leakage is presented. The algorithm was compared favorably in speed with a previous one based on discrete Fourier transforms.

Introduction

The lack of synchronization is the main factor responsible for the leakage error in the Fast Fourier Transform (FFT) [1] or the FHT [2].

The errors that appear in the FFT/FHT results are referred to as leakage, whereas in the time domain this same error is referred to as truncation error. In both cases the error arises because the sampling is not synchronized with the signal being sampled. An optimized algorithm that turns negligible the truncation error has already been developed [3]. Several technical details about that algorithm are discussed in [4]. Its drawback is the computation time required to evaluate the discrete Fourier transform.

The contribution of this paper is to modify the algorithm described in [3] for accurately evaluating the FHT coefficients of arbitrary voltage signals.

Algorithm

Codes for fast computing the discrete Hartley transform (DHT) are easily available as built-in functions of most current laboratory software packages. The operation is performed in place and the input data array is overwritten which implies in memory savings. The FHT is as fast as or faster than the FFT [2]. The FHT makes it possible to dispense with imaginary numbers in the computation of Fourier transforms. Since the data is real, the FHT output is also real.

The algorithm uses the same 'Samp_parm' routine described in [5] to optimize the burst time Nt_{samp} for a given signal frequency f_0, number of harmonics m, number of bursts n and a few other constant default parameters. The routine outputs a number of samples $N = 2m$ when $n = 4m$ and m is an even number within the range 52-140 for $f_0 = 60$ Hz (or within 52-146 for $f_0 = 50$ Hz). When using the FFT/FHT, it is convenient to choose the number of sample points according to $N = 2^{\gamma}$, where γ is an integer. Therefore, it is possible to measure 64 or 128 harmonics of a signal at power frequencies.

The digital sampling voltmeter (DSV) is configured in the same way as described in [5], except for the trigger level that is now selected arbitrarily by the user so that the fundamental frequency measurement can be done by counting level crossings at less noisy segments of the signal.

The algorithm transfers each burst data from the DSV to a computer where the FHT is evaluated. The real and imaginary parts of the FFT at the k-th burst are directly obtained as the even and odd parts of the FHT of the corresponding burst, that is

$$a_{jk} = \frac{x_{jk} + x_{(N-j)k}}{2}, \tag{1a}$$

$$b_{jk} = \frac{x_{jk} - x_{(N-j)k}}{2} \tag{1b}$$

where $j = 1, \ldots, m$. The harmonic magnitudes and phase angles at each burst are obtained from

$$V_{jk} = \left[\left(a_{jk}^2 + b_{jk}^2\right)/2\right]^{1/2} \tag{2a}$$

$$\theta_{jk} = \tan^{-1}\left(-b_{jk}/a_{jk}\right) \tag{2b}$$

The average of a_{jk} and b_{jk} over all bursts is then calculated as

$$\bar{a}_j = \frac{1}{4m}\sum_{k=0}^{n-1} V_{jk}\cos\theta_{jk} \tag{3a}$$

$$\bar{b}_j = -\frac{1}{4m}\sum_{k=0}^{n-1} V_{jk}\sin\theta_{jk} \tag{3b}$$

and the resulting estimates $V_{j\mathrm{e}}$, $\theta_{j\mathrm{e}}$ for the harmonic parameters are obtained from (2) by substituting $\bar{a}_j$, $\bar{b}_j$ for a_{jk}, b_{jk} (the magnitudes should be doubled as we are only interested in the spectrum for positive frequencies). These harmonic parameters are finally referred to the fundamental and corrected for all known DSV systematic effects, that is

$$d_{j\mathrm{e}} = \frac{k_{\mathrm{bw}}(jf_0)\cdot k_{\mathrm{int}}(jf_0)\cdot V_{j\mathrm{e}}}{k_{\mathrm{bw}}(f_0)\cdot k_{\mathrm{int}}(f_0)\cdot V_{1\mathrm{e}}} \tag{4a}$$

$$\gamma_{j\mathrm{e}} = \theta_{j\mathrm{e}} + j\left(\theta_{\mathrm{R}} - \theta_{1\mathrm{e}}\right) \tag{4b}$$

where $k_{\mathrm{bw}}(jf_0)$ and $k_{\mathrm{int}}(jf_0)$ are the frequency response corrections of the DSV input stages and the integrating A/D converter at the j-th harmonic (see

[3]), respectively, and θ_R is an arbitrary reference angle. Equation (4b) assumes that the DSV input stages can be modeled as a linear-phase filter. Note that the correction κ of the DSV dc voltage mode error (see [3]) was cancelled in (4a). One does not need to care about the non negligible A/D converter gain errors when evaluating the harmonic magnitudes as a percentage of the fundamental.

Performance tests

A stable, high-resolution DSV was used to measure the harmonic magnitudes and phase angles of a periodic signal generated by a stable, digitally-synthesized, arbitrary signal generator (also used in [3]). The DSV was sequentially controlled by two algorithms: (i) the FHT-based one and (ii) a one-channel version of that described in [3].

Several nonsinusoidal signals in the 10 V range were synthesized and separately applied to the DSV input. Table I shows the results for both algorithms ((i) and (ii)) when a 60-Hz half-wave rectified signal was measured by assuming $m = 64$. Only even harmonics are shown since the odd harmonic magnitudes are small compared to the fundamental. Due to space limitations, we report only the results for the first 16 harmonics. The differences between the results are well within the measurement uncertainty $u(d_{je})$. The standard deviation $s(d_{je})$ of the mean (same for both algorithms) is an indication of the stability.

Table I. Comparison of harmonic magnitudes.

Har No.	Magn. (%) (using [3])	Magn. (%) (using FHT)	$u(d_{je})$ (10^{-6})	Diff. (10^{-6})	$s(d_{je})$ (10^{-6})
1	100	100	–	–	–
2	42.47780	42.47777	5.9	−0.3	1.2
4	8.49698	8.49696	5.4	−0.2	0.8
6	3.63891	3.63879	5.4	−1.2	0.3
8	2.02127	2.02115	5.4	−1.2	0.3
10	1.28745	1.28734	5.4	−1.1	0.2
12	0.89443	0.89431	5.4	−1.2	0.3
14	0.65335	0.65323	5.4	−1.2	0.2
16	0.49962	0.49955	5.4	−0.7	0.2

Table II. Comparison of measuring times.

m	Aperture time [(1)] (ms)	Meas. Time (using [3])	Meas. Time (using FHT)
64	0.1003	60 s	35 s
128	0.0352	3 min 40 s	1 min 23 s

(1) $t_{aper} = t_{samp} - 0.00003$ s [5].

It is assumed that the uncertainty associated with the results is the same for both algorithms. In order to minimize the running time, the uncertainty analysis is only made by the algorithm (ii). The uncertainties associated with d_{je} and γ_{je} are

$$u(d_{je}) \approx u(V_{je}) / \sqrt{2} V_{1e} \qquad (8a)$$

$$u(\gamma_{je}) = \left[u^2(\theta_{je}) + j^2 u^2(\theta_{1e})\right]^{1/2} \qquad (8b)$$

where $u(\theta_{je}) = u(V_{je})/\sqrt{2}V_{je}$ and $u^2(V_{je})$ is evaluated from equation (3) in [3].

It was also verified that the FHT-based algorithm is capable to measure 128 harmonic magnitudes as a percentage of the fundamental of half-wave rectified signals at power frequencies (with about 44% THD), with an uncertainty of less than 5 parts in 10^6 relative to the fundamental.

It is of interest to know how the FHT-based algorithm compares in speed with that based on the discrete Fourier transform. A computer with 166-MHz clock was used to obtain the results in Table II. Though the data acquisition and transfer require some fixed time to be accomplished, further reduction in measuring time is to be expected with faster computers.

Conclusions

It was verified that an algorithm based on Fast Hartley Transform and Swerlein's algorithm can be used to accurately measure 64 or 128 harmonic magnitudes as a percentage of the fundamental of distorted signals at power frequencies with acceptable measuring times. Evidences show that it is possible to neglect the leakage error with this algorithm. The algorithm was compared favorably in speed with a previous one based on discrete Fourier transforms.

The measurement uncertainty would be even less if one assumes $m = 256$, but unfortunately the 'Samp-parm' routine in [5] does not output a power-of-two number of samples at power frequencies when inputted with $n = 4{\times}256 = 1024$ bursts. Changes have been made in the routine to solve this problem, but they resulted in degraded measurement uncertainties. Anyway, this topic deserves further investigation.

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