EVALUATION OF UNCERTAINTY IN AC VOLTAGE MEASUREMENT USING A DIGITAL VOLTMETER AND SWERLEIN´S ALGORITHM *

G.A. Kyriazis[1], R. Swerlein[2]

Instituto Nacional de Metrologia, Normalização e Qualidade Industrial[1]
Duque de Caxias, RJ, 25250-020 Brazil
Agilent Technologies[2]
Loveland, Colorado 80537 USA

Abstract

A model is proposed for a sampling algorithm that uses a high-resolution voltmeter for measuring the RMS value of a sinusoidal voltage waveform. The uncertainty associated with the result of measurement is evaluated according to the rules in the ISO/BIPM *Guide to the Expression of Uncertainty in Measurement*.

Introduction

About ten years have passed since the introduction of the Swerlein´s algorithm. It was developed for the accurate measurement of RMS voltage at low frequencies using an HP 3458A digital voltmeter [1]. The algorithm has been exhaustively tested. Differences of less than $1.2 \cdot 10^{-6}$ between it and a multi-junction thermal converter were reported for voltages in the 10-V range and frequencies in the 10-100 Hz range [2]. Differences of less than $10 \cdot 10^{-6}$ were also reported for voltages in the range 0.2-20 V and frequencies up to 1 kHz [3]. The algorithm has been extensively used as a standard in industry.

The contribution of this paper is to provide an evaluation of uncertainty in measurement according to the ISO/BIPM *Guide to the Expression of Uncertainty in Measurement* [4]. A model for the algorithm is presented and the uncertainty contributions are evaluated.

Model

The algorithm samples a member of the *ensemble* of a random process $V(t)$ at times t_i, $i = 1, 2, \ldots, N \cdot N_{burst}$. Considering the samples as random variables $V_i = V(t_i)$, estimates for the expectation, variance and power in the process are, respectively,

$$\overline{V} = \frac{1}{N \cdot N_{burst}} \cdot \sum_{i=1}^{N \cdot N_{burst}} V_i , \qquad (1)$$

$$S^2 = \frac{1}{N \cdot N_{burst}} \cdot \sum_{i=1}^{N \cdot N_{burst}} \left(V_i - \overline{V}\right)^2 \qquad (2)$$

$$\hat{P} = S^2 + \overline{V}^2 . \qquad (3)$$

An estimate of the DC component is obtained from the sample average. An estimate of the RMS value (without the DC component) is evaluated from the central moment of order 2 of the sample. The RMS value results from the estimate of the square root of the power in the random process.

The model used for the measurement of the RMS value (without DC component) is

$$V_{ACRMS} = \sqrt{\frac{1}{N \cdot N_{burst}} \cdot \sum_{i=1}^{N \cdot N_{burst}} \left(V_i - \overline{V}\right)^2} + R \qquad (4)$$

where V_i is the sample corrected for all the known systematic effects and R is equivalent to the difference between the indication that would be obtained with an ideal A/D converter (not limited by resolution) and the indication of a real integrating A/D converter (IADC).

After being applied to input terminals, the signal is conducted to a passive signal conditioner. In the 1 V and 10 V ranges, the conditioner is a unit-gain low-pass filter whose response falls at 20 dB/decade above 120 kHz. The filter output impedance is 10 kΩ. The signal is then applied to an active amplifier. In the 1 V and 10 V ranges, the amplifier gain is 10 and 1, respectively, and its bandwidth is greater than 1 MHz. If the voltmeter is configured for the 100 V and 1000 V ranges, the passive signal conditioner is a resistive divider with 100:1 ratio composed of a 10-MΩ resistor and a 100-kΩ resistor. Printed circuit board capacitance and dissipation factor make it difficult an evaluation of the frequency response above 1 kHz. For low frequencies, the frequency response can be modeled as a term that falls at 20 dB/decade above 36 kHz. Finally, the signal is applied to the IADC. In the time-domain, the response is an average of the input signal over a time interval equivalent to the aperture time specified by the user. This average value is generally different from the waveform value at the middle of the aperture time interval *Aper*. The relation between the signal value at time t_i and the uncorrected sample V_i' is,

$$V_i = K_f \cdot K_{Aper} \cdot K \cdot K_a \cdot V_i' \qquad (5)$$

* Unpublished extended version of the paper at *CPEM Digest*, Ottawa, June 16-21, 2002, pp. 24-25.

where K_f is the frequency response correction (finite bandwidth) of both the passive signal conditioner and the active amplifier, K_{Aper} is the IADC frequency response correction (finite aperture time), K is the correction of the DC voltage mode error, and K_a is the correction of the error introduced by the input attenuator. Substituting (5) in (4),

$$V_{ACRMS} = K_f \cdot K_{Aper} \cdot K \cdot K_a \cdot S' + R \qquad (6)$$

where,

$$S' = \sqrt{\frac{1}{N \cdot N_{burst}} \cdot \sum_{i=1}^{N.N_{burst}} \left(V_i' - \overline{V}'\right)^2}$$

$$\overline{V}' = \frac{1}{N \cdot N_{burst}} \cdot \sum_{i=1}^{N \cdot N_{burst}} V_i'$$

Assuming independent quantities, the square of the relative standard uncertainty associated with the measurement result can be approximated as

$$u^2_{V_{ACRMS}} \Big/ V^2_{ACRMS} \approx u^2_{K_f} + u^2_{K_{Aper}} + u^2_K + u^2_{K_a} + \\ + u^2_{V_i'} \Big/ V^2_{ACRMS} N_{burst} N + u^2_R \Big/ V^2_{ACRMS} \qquad (7)$$

Numerical values for the relative standard uncertainty at several frequencies and voltage ranges are listed in Table I. They were evaluated from the uncertainty contributions related in the next section. It is assumed that the signal is generated by an ideal voltage source.

Table I. Relative standard uncertainty (parts in 10^6).

	Frequency (Hz)			
Range (V)	1	10	100	1000
1 / 10	2 / 1.8	2 / 1.8	2.5 / 2.3	30
100 / 1000	2.4 / 2.3	2.4 / 2.3	4.1	230

Uncertainty Contributions

Conditioner and Amplifier Frequency Response Correction

Based on the previous description of the input conditioner and amplifier, this correction is

$$K_f = \sqrt{1 + \left(F / F_{BW}\right)^2} \qquad (8)$$

where F is the signal frequency and F_{BW} is the cut-off frequency (120 kHz @ 1 V/ 10 V ranges and 36 kHz @ 100 V/ 1000 V ranges). These responses model the voltmeter behavior with a relative standard uncertainty associated with the cut-off frequency of $u_{F_{BW}} / F_{BW} = 30\%$ [1]. Taylor expanding (8),

$$K_f \approx 1 + \delta V / V = 1 + \left(F / F_{BW}\right)^2 / 2 \qquad (9)$$

The square of the standard uncertainty associated with this correction is

$$u^2_{K_f} = u^2_{\delta V / V} = \left(\delta V / V\right)^2 u^2_{\delta V / V} \Big/ \left(\delta V / V\right)^2 \qquad (10)$$

where, neglecting the relative standard uncertainty associated with the time base (0,01%),

$$u^2_{\delta V / V} \Big/ \left(\delta V / V\right)^2 = 4 u^2_{F_{BW}} \Big/ {F_{BW}}^2 \qquad (11)$$

IADC Amplitude and Time Base Resolution

The IADC resolution is equivalent to the relative difference between the algorithm indication that would be obtained by an ideal A/D converter, not limited by amplitude resolution R_A and time base resolution Δ, and the indication of the real IADC. It can be modeled as

$$R = R_A + \Delta \,, \qquad (12)$$

The relative difference between the algorithm indication resulting from a burst of the real IADC, i.e.,

$$V_{ACRMS} = \sqrt{\frac{1}{N} \cdot \sum_{i=1}^{N} A^2 \cdot \sin^2\left(2\pi F t_i\right)} \,, \qquad (13)$$

and the indication ($A / \sqrt{2}$) of an ideal A/D converter not limited by resolution can be approximated for large N by

$$\Delta \approx \frac{\sin\left(2\pi F \cdot N \cdot T_{samp}\right)}{4\pi F \cdot N \cdot T_{samp}} \cdot \sin\left(4\pi F t + \Phi\right) \qquad (14)$$

In order to reduce the measurement time, the algorithm tries to take a burst of N samples spaced of T_{samp}, where $N \cdot T_{samp}$ is an integer number of periods, so that the numerator of (14) becomes null. In practice, however, due to the 100-ns time base quantization of the voltmeter [5], for a given number of samples, the total sampling time can differ up to 100N ns from an integer number of periods. This difference reaches a maximum when $10^{-7}N = T_{samp}$, as it is always possible to choose N so that the difference between the total sampling time and an integer number of periods is less than T_{samp}. Thus, as T_{samp} is small, (14) can be written as

$$\Delta \le \frac{10^{-7}}{2T_{samp}} \cdot \sin\left(4\pi F t + \Phi\right) \;,\; 10^{-7}N < T_{samp} \qquad (15)$$

$$\Delta \le \frac{1}{2N} \cdot \sin\left(4\pi F t + \Phi\right) \;,\; 10^{-7}N \ge T_{samp} \qquad (16)$$

It should be noted from (15) that the sampling period should be as large as allowed by the required bandwidth, that can be defined as $N_{harm} \cdot F$, where N_{harm} is the minimum number of waveform harmonics that will be sampled without *aliasing* (typically N_{harm} = 6). The maximum sampling period should attend the sampling theorem, i.e., $T_{samp} < 1/2N_{harm}F$. If it does not, the algorithm increases the sampling frequency in order to assure the *alias* occurrence exactly at $N_{harm} \cdot F$. The manufacturer specifies the maximum aperture time for a given sampling period in order to prevent trigger-too-fast errors [5]. The algorithm takes this into account and estimates the aperture time as $Aper = T_{samp} - 3 \cdot 10^{-5}$ s.

The IADC amplitude resolution decreases with the aperture time. In order to preserve a minimum amplitude resolution of 6½ digits (21 bits), the selected aperture time should lie between 100 μs and 10 ms [5]. The voltmeter accuracy is also degraded as the aperture time decreases, so that a target aperture time of 0.001 s is typically selected. Due to the above bandwidth requirement the actual aperture time is kept almost constant up to $F = 1/2N_{harm}T_{samp}$ where it starts decreasing with the signal frequency as $Aper = 1/2N_{harm}F - 3 \cdot 10^{-5}$ s. Eventually, it becomes less than 100 μs so that an amplitude resolution of 5½ digits (18 bits) is observed in the high frequency range. The probability density function (*pdf*) associated with R_A is rectangular.

The algorithm also minimizes the ripple that depends on the initial sampling time in (14). The internal level trigger of the voltmeter is used to start a burst delayed T_D from the waveform null-crossing. The difference between the indications of the ideal and real converter is thus “frozen” in time in a value

$$\Delta \approx Min\left(\frac{10^{-7}}{2T_{samp}}, \frac{1}{2N}\right) \cdot sin(4\pi F T_D + \Phi) \qquad (17)$$

If N_{burst} bursts of N samples are taken delayed from each other by a time interval $T_D = 1/N_{burst}F$, and the average of the N_{burst} results is evaluated, the “frozen” limits (for each value of T_D) will be cancelled. Assuming a conservative limit of Δ/20 for this cancellation,

$$\Delta \approx Min\left(10^{-8} / 4T_{samp}, 1/40N\right) \qquad (18)$$

The *pdf* associated with Δ is also rectangular.

The ripple frequency in (17) is $2F$. The sampling theorem implies a maximum value for T_D of $1/4F$, i.e., a minimum number of 4 (four) bursts, if the waveform is to be sampled over a period. If the input signal has harmonics, then the ripple will have higher frequency components, and this will require smaller delays, i.e., a bigger number of bursts. It can be shown that the distortion introduced by a third harmonic generates ripple additional components of frequencies $2F$ and $4F$ and that the amplitudes of these components, for a 1% distortion, are much smaller than the main ripple [1]. As the algorithm assumes a pure sinusoidal waveform, there is no need to work with more than 6 (six) bursts.

IADC Frequency Response Correction

The relation between the average value of the input signal over the selected aperture time and the uncorrected sample V_i' is

$$M_i = K_f \cdot K \cdot K_a \cdot V_i' \qquad (19)$$

where the average value M_i is

$$M_i = \frac{1}{Aper} \cdot \int_{t_i - Aper/2}^{t_i + Aper/2} V(t) \cdot dt \qquad (20)$$

In the specific case of a pure sinusoidal waveform,

$$M_i = \frac{sin(\pi \cdot F \cdot Aper)}{\pi \cdot F \cdot Aper} \cdot V_i \qquad (21)$$

Substituting (23) in (21) and comparing with (5), the IADC frequency response correction is

$$K_{Aper} = \left(\frac{sin(\pi \cdot F \cdot Aper)}{\pi \cdot F \cdot Aper}\right)^{-1} \qquad (22)$$

Differently from all corrections, this one is significant, i.e., for a 1 ms aperture time and 100 Hz fundamental frequency, the correction is nearly 2%. However, this error may be accurately corrected. Taylor expanding (22),

$$K_{Aper} \approx 1 + \delta V / V = 1 + (\pi \cdot F \cdot Aper)^2 / 3! \qquad (23)$$

The square of the standard uncertainty associated with K_{Aper} is given by an expression similar to (10). As the clock used for the sample timing is also used to establish the time in the frequency measurement, it is sufficient to consider the relative standard uncertainty associated with the aperture time, i.e.,

$$u^2_{\delta V/V} / (\delta V / V)^2 = 4u^2_{Aper} / Aper^2 \qquad (24)$$

where, $u_{Aper}/Aper$ is evaluated from the relative standard uncertainty associated with the time base (0.01%) and the aperture time resolution R_T (100 ns or 0.01%·*Aper*). The *pdf* associated with R_T is rectangular.

It should be observed that a great amount of the uncertainty associated with the measurement of RMS value of distorted waveforms is related to the fact that (22) corrects the IADC frequency response only for the fundamental frequency of a distorted waveform. The algorithm assumes a pure sinusoidal waveform. No attempt is made to evaluate the consistency of this assumption.

DC Voltage Mode Error Correction

The correction of the DC voltage mode error can be modeled as

$$K = K_{DC} \cdot K_L \cdot K_G, \tag{25}$$

where K_{DC} is the correction of the error listed in a calibration certificate, K_L is the correction of the linearity of the instrument and K_G is the correction of the IADC gain error for aperture times less than 1 s. The manufacturer states the error limits for each voltage range and calibration period [5]. The 24-h basic accuracy is chosen in this paper. This accuracy is based on a aperture time equivalent or greater than 1 s and is degraded for smaller aperture times. The limits of the gain error are also provided by the manufacturer. The *pdf* associated with K_L is rectangular (idem for K_G).

Input Attenuator Error Correction

In the high voltage ranges, the input signal passes through an attenuator with a 100-kΩ output resistance. In the path to the amplifier, the signal faces a 35-pF capacitance of low dissipation factor (FET inputs, ceramic capacitors, etc) and about 15-pF printed circuit board capacitance of high dissipation factor. In the low voltage ranges, the input signal is conducted to a 10 kΩ resistor whose output is applied to about 120-pF capacitance and about 15-pF printed circuit board capacitance.

A capacitor C that has a dissipation factor D_{ef} acts as it had a resistance in parallel equivalent to $1/2\pi FCD_{ef}$. This resistance creates a resistive divider with the attenuator output resistance R_o. The correction of the error caused by this divider is

$$K_a \approx 1 + \delta V / V = 1 + 2\pi F R_o C D_{ef} \tag{26}$$

This correction was not implemented in the original program [1]. The evaluated error was then regarded as an uncertainty contribution. The square of the standard uncertainty associated with this correction is given by an expression similar to (10). Neglecting the relative standard uncertainty associated with the time base,

$$u^2_{\delta V/V} \big/ (\delta V / V)^2 = u^2_{R_o} \big/ R_o^2 + u^2_C \big/ C^2 + u^2_{D_{ef}} \big/ D^2_{ef} \tag{27}$$

The printed circuit board dissipation factor is known to vary randomly from one batch to the next according to a frequency distribution that is not gaussian. The lowest observed low frequency dissipation factor is 0.2% and the highest is 10%. The printed circuit board capacitance has a typical dissipation factor of 0.4%. Thus, for the high voltage ranges, the best estimate d_{ef} of the effective dissipation factor of the 50-pF combined capacitance is 0.12% (= 0.4% x 15/50) and its value is between the limits a_- = 0.06% and a_+ = 3%. As the best estimate is not in the middle of the interval, the *pdf* cannot be uniform throughout the interval [4]. Based on the Principle of Maximum Entropy [6], it can be shown that the *pdf* in the asymmetric case has a variance

$$u^2_{D_{ef}} = b_+ b_- - (b_+ - b_-)/\lambda \tag{28}$$

where the lower limit is written as $a_- = d_{ef} - b_-$ and the higher limit as $a_+ = d_{ef} - b_+$. The relative standard uncertainty associated with the effective dissipation factor is 50%. The uncertainties associated with the attenuator capacitance and output resistance are evaluated as 20% and 5%, respectively.

Noise

The standard uncertainty associated with each uncorrected sample can be evaluated from the information about the noise in the individual samples. The RMS value of the noise is provided by the manufacturer [5].

Conclusion

A model was proposed for the Swerlein´s algorithm. An evaluation of the standard uncertainty associated with the algorithm estimate was presented. The standard uncertainty was evaluated to be less than $5 \cdot 10^{-6}$ in the 1-1000 V and 1-100 Hz ranges. The uncertainty contribution associated with the conditioner and amplifier frequency response correction is dominant in the high frequency range.

References

[1] R. L. Swerlein., "A 10 ppm Accurate Digital AC Measurement Algorithm". In: Proc. NCSL, pp. 17-36, Albuquerque, USA, Aug. 1991.

[2] M. Kampik, H. Laiz, M. Klonz, "Comparison of Three Accurate Methods to Measure AC Voltage at Low Frequencies", IEEE Trans. Instrum. Meas., Vol. 49, No. 2, pp. 429-433, Apr. 2000.

[3] R. Lapuh, A. Arnsek, I. Visocnik, et al., "Evaluation of Low Frequency AC Voltage Measurement Using Integrating Sampling Technique", IMTC Digest, 1999.

[4] ISO/BIPM, *Guide to the Expression of Uncertainty in Measurement*, Geneva:International Organization for Standardization, 1995.

[5] Hewlett-Packard, *HP 3458A Operating Manual*.

[6] W. Wöger, "Probability Assignment to Systematic Deviations by the Principle of Maximum Entropy", IEEE Trans. Instrum. Meas., Vol. 36, no. 2, pp. 655-658, Jun. 1987.